Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals

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Joint work with Tom Dupré La Tour., Mainak Jas, Alexandre Gramfort, Cédric Allain, Lindsey Power, Tim Bardouille

Context: functional Neuroimaging

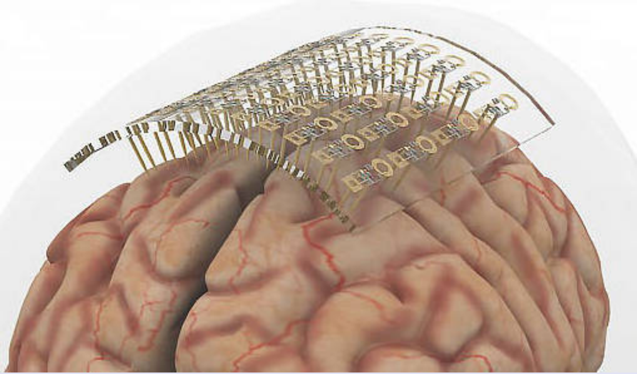
Goal: Study the brain mechanisms while it is functioning.

Outputs:

- **Functional Atlases:** Link areas of the brain to specific cognitive functions.
- **Functional Connectivity:** Highlight the information flow in the brain.
- **Healthcare:** Develop bio-markers for neurological disorders.

How to record living brains electrical activity: **Electrophysiology**

Direct measurement: intracranial EEG.



High Localization

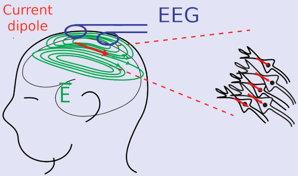
Low Resolution

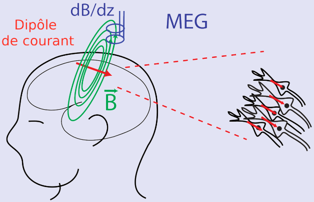
Invasive

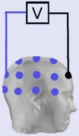
Context: functional Neuroimaging

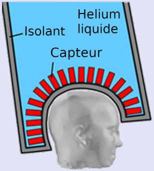
How to record living brains electrical activity: **Electrophysiology**

Remote measurement: M/EEG.



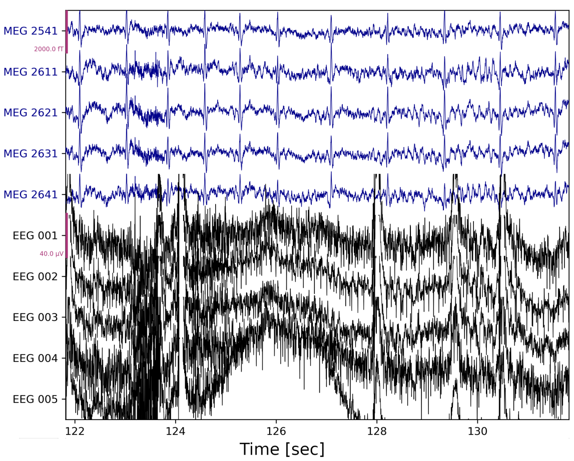






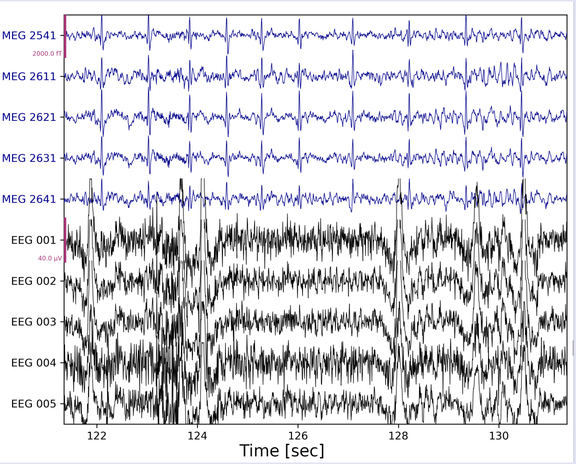
No Localization **Global** **Non Invasive**

Multivariate time-series X



Noisy **Many artifacts** **Complex**

Multivariate time-series X



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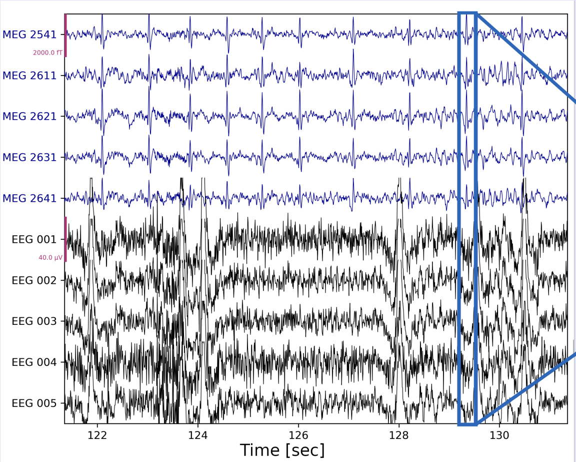

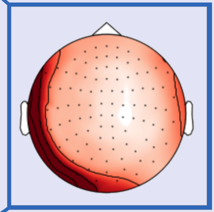

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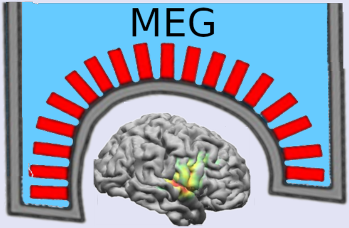
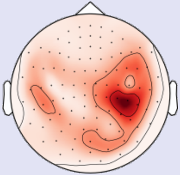


$\xrightarrow[\text{Equations}]{\text{Maxwell's}}$

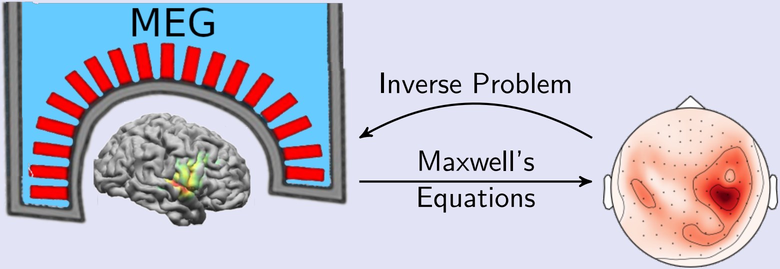


$\boxed{\varepsilon}$ $\boxed{G}$ $\boxed{X}$

Electrical activity **Observed signal**

Forward model: $X = G\varepsilon$

How to get back to electrical activity?



$\boxed{\varepsilon}$ **Electrical activity**

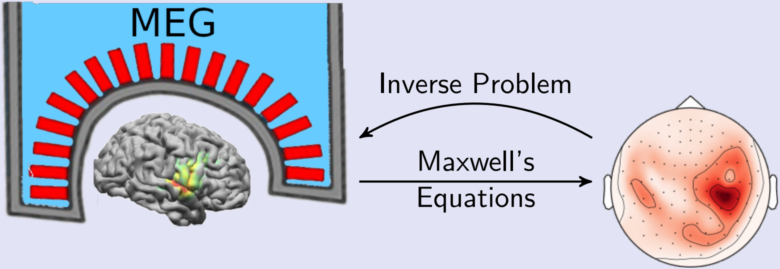
$\boxed{G}$

$\boxed{X}$ **Observed signal**

Forward model: $X = G\varepsilon$

Inverse problem: $\varepsilon = f(X)$ (ill-posed)

How to get back to electrical activity?



$\boxed{\varepsilon}$ **Electrical activity**

$\boxed{G}$

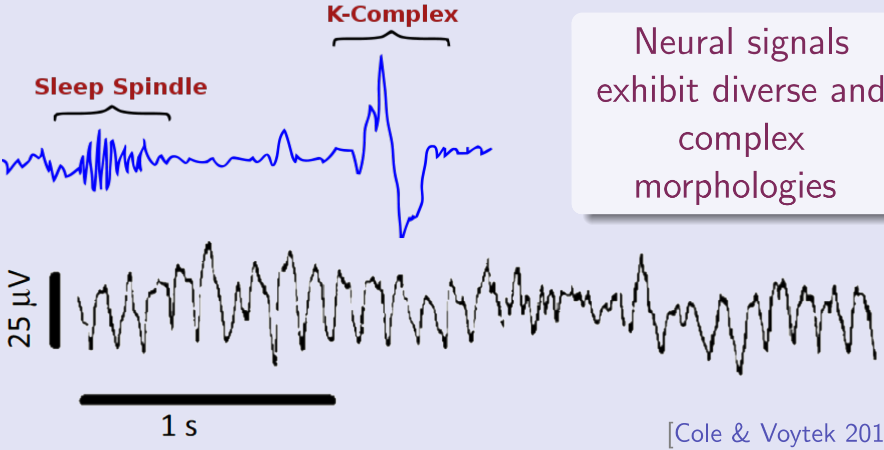
$\boxed{X}$ **Observed signal**

Forward model: $X = G\varepsilon$

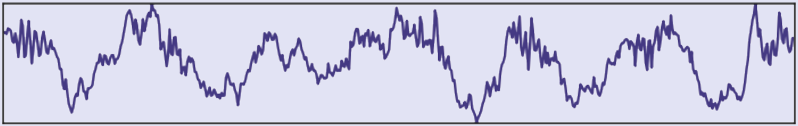
Inverse problem: $\varepsilon = f(X)$ (ill-posed)

- Dipole fit [Sarvas, 1987]
- Regularized optimization [Gramfort et al., 2012]
- Deep-learning [Hecker et al., 2021]

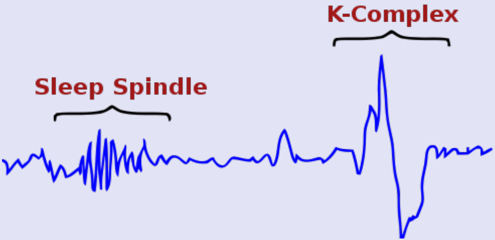
Neural signals exhibit diverse and complex morphologies



[Cole & Voytek 2017]



[Dupré la Tour et al. 2017]



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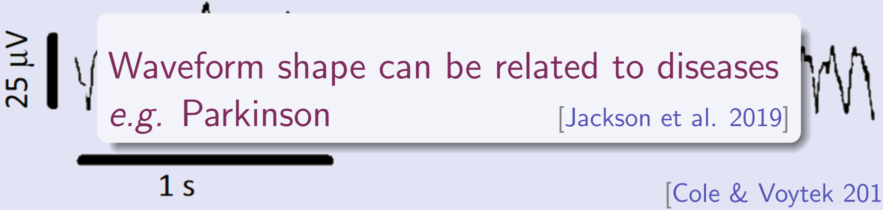
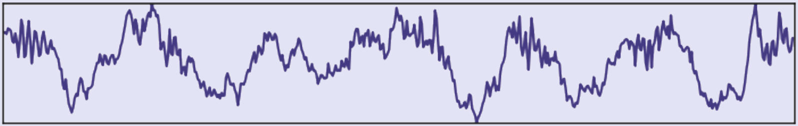


Waveform shape can be related to diseases
e.g. Parkinson [Jackson et al. 2019]

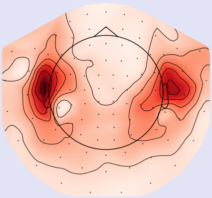
[Cole & Voytek 2017]

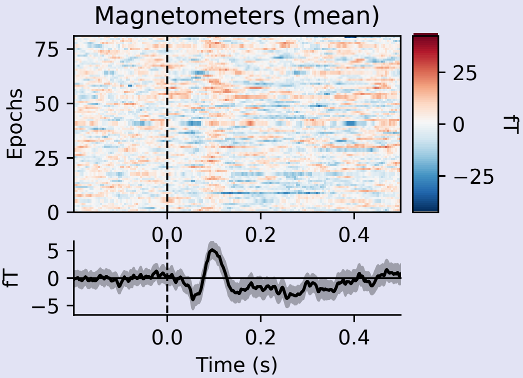


[Dupré la Tour et al. 2017]

- Subject is presented some stimuli – Audio, Visual, Motor, ...
- Record onset of the stimuli
- Average signal on window aligned around the stimulus

Evoked response to an auditory stimuli





[MNE-Python]

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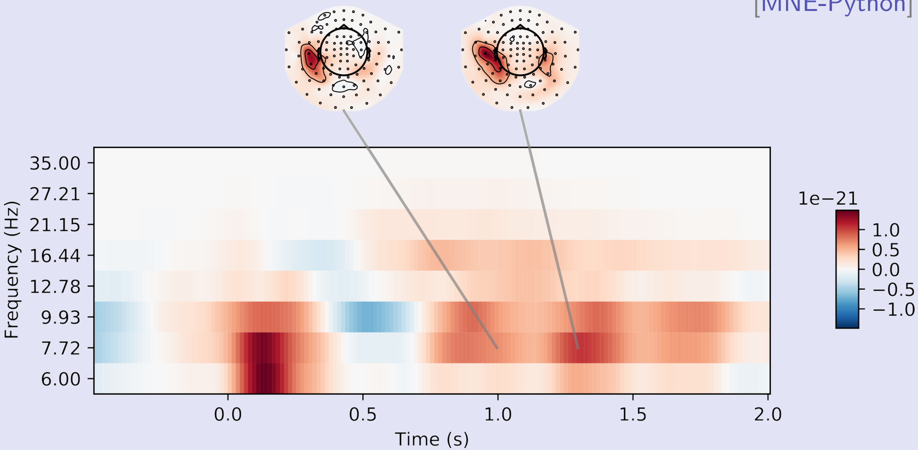
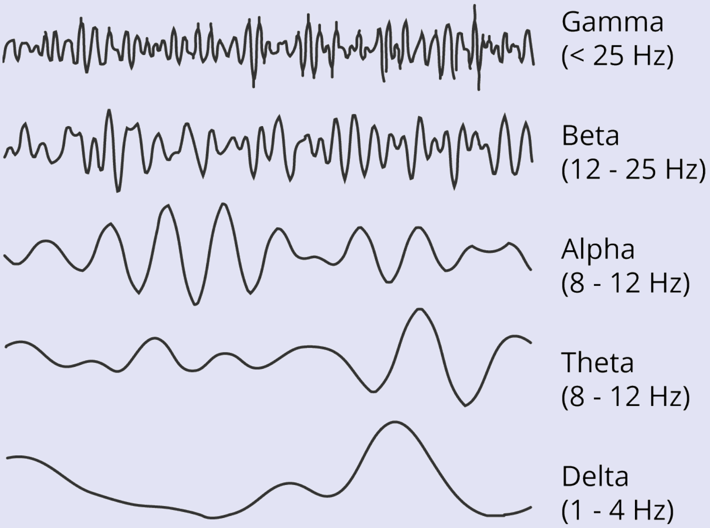
- Average PSD on window aligned around the stimulus

[MNE-Python]



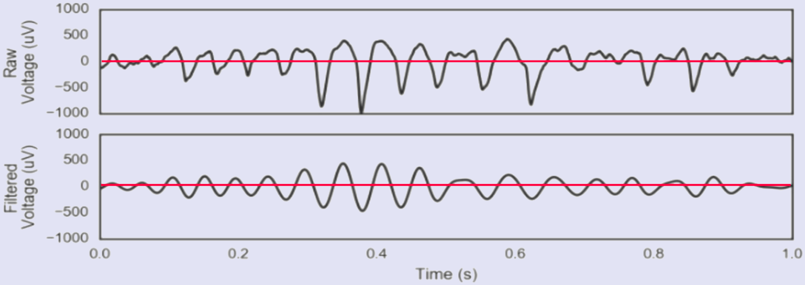
Evoked response to an somatosensory stimuli

“Textbook” brain rythms



Linear filtering

After Linear filters, everything looks like a sinusoïd.



⇒ Lose the asymmetry and the shape information

Fourier Fallacy

"Even though it may be possible to analyze the complex forms of brain waves into a **number of different sine-wave** frequencies, this may lead only to what might be termed a "**Fourier fallacy**", if one assumes **ad hoc** that all of the necessary frequencies actually occur as periodic phenomena in **cell groups** within the brain."

[Jasper, 1948]

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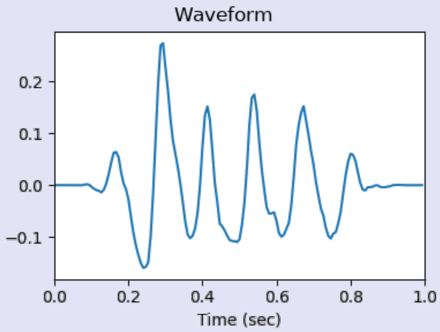

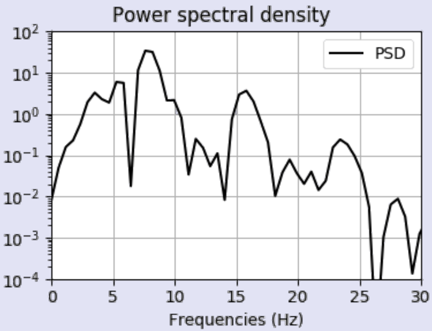

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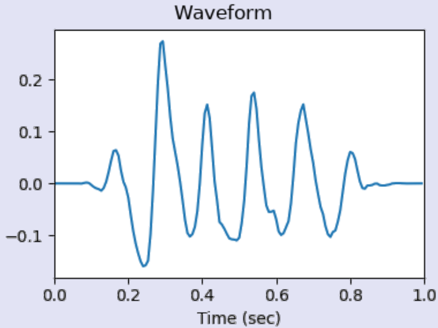
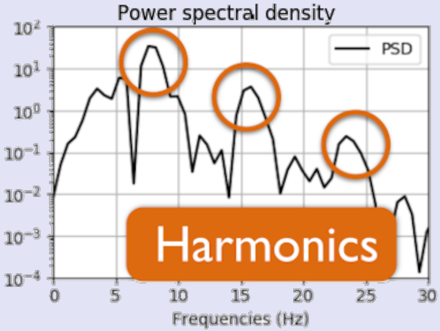




Learning the waveform: Convolutional Dictionary Learning

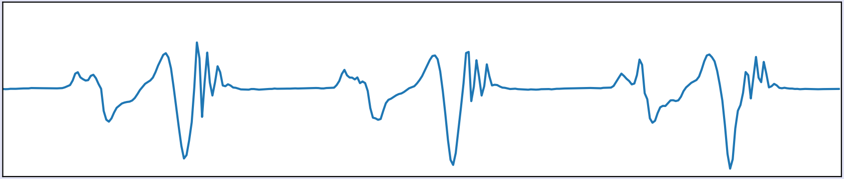
References

- Grosse, R., Raina, R., Kwong, H., and Ng, A. Y. (2007). Shift-Invariant Sparse Coding for Audio Classification.

  Cortex, 8:9

Local structure in signals

x^n



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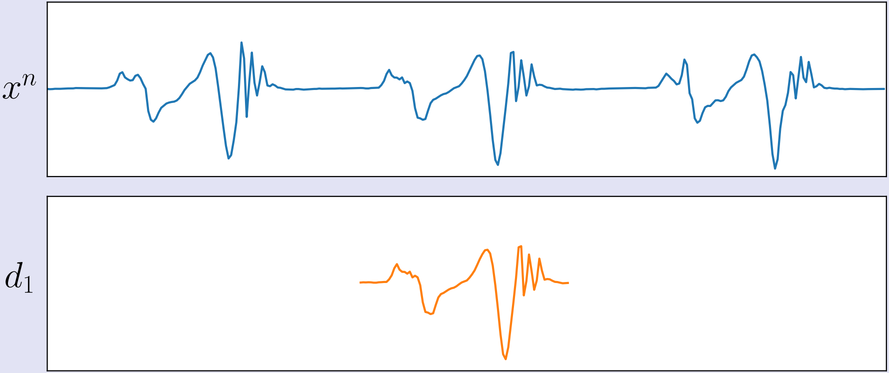

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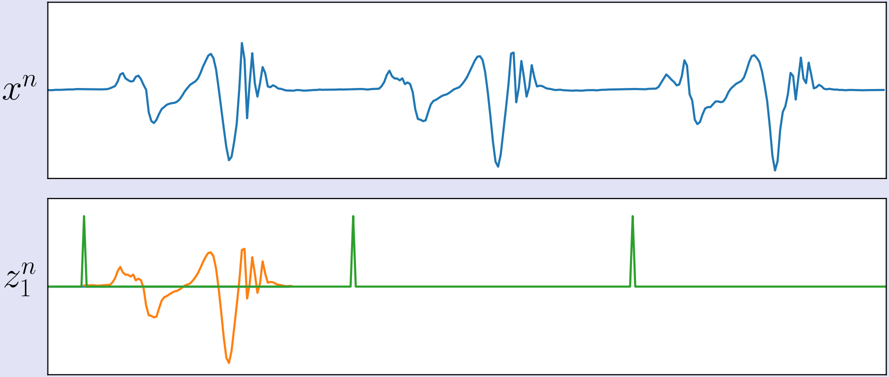

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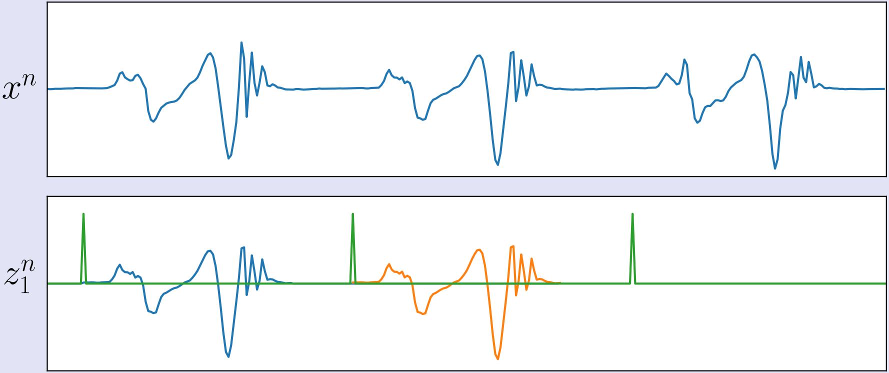
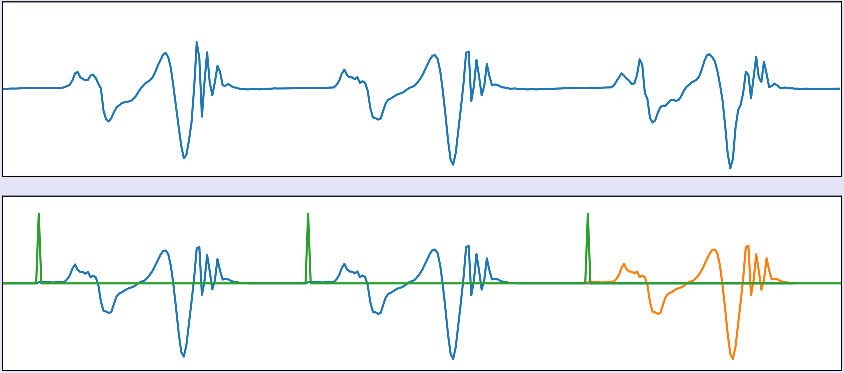


Local structure in signals

Key idea: decouple the localization of the patterns and their shape

x^n

z_1^n



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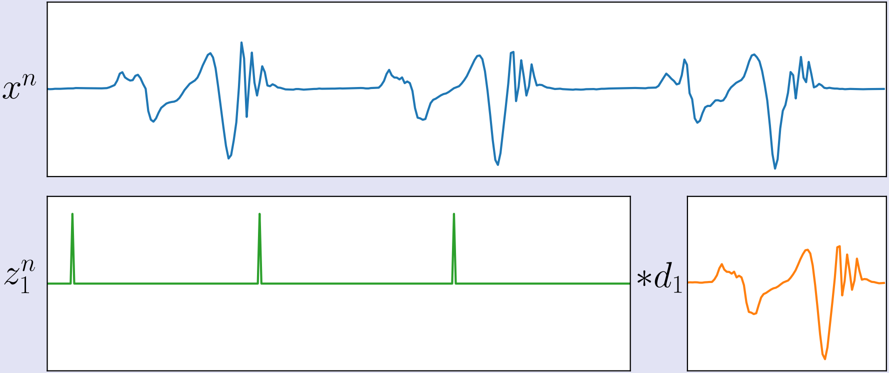
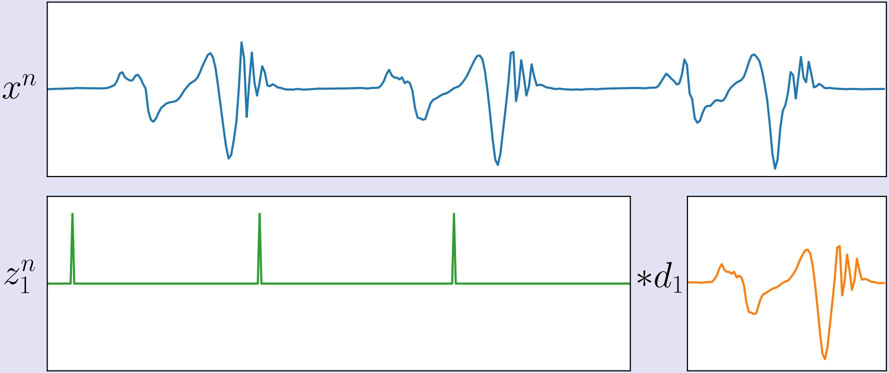


Key idea: decouple the localization of the patterns and their shape



$$\boxed{x^n}[t] = \sum_{k=1}^{K} (\boxed{z_k^n} * \boxed{d_k})[t] + \varepsilon[t]$$

For a set of N univariate signals x^n, solve

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| \boxed{x^n} - \sum_{k=1}^{K} \boxed{z_k^n} * \boxed{d_k} \right\|_2^2 + \lambda \sum_{k=1}^{K} \|\boxed{z_k^n}\|_1,$$
$$\text{s.t.} \quad \|\boxed{d_k}\|_2^2 \leq 1$$

Hypothesis: patterns d_k are not present everywhere in the signal. They are localized in time.

$\Rightarrow$ Sparse activation signals z

Technical hypothesis: the patterns are in the ℓ_2-ball: $\|d_k\|_2^2 \leq 1$.

Optimization strategy

Bi-convex: The problem is not jointly convex in z_k^n, and d_k but it is convex in each block of coordinate.

Alternate minimization (*a.k.a.* Bloc Coordinate Descent):

- **Z-step:** given a fixed estimate of the atom, compute the activation signal z_k^n associated to each signal x^n.
- **D-step:** given a fixed estimate of the activation, update the atoms in the dictionary d_k.

Unrolled optimization:

- **Z-step:** use an fixed differentiable procedure $f(x^n, D)$.
- **D-step:** learn D through back-propagation.

[Malezieux et al. 2022]

How to extend CSC to multivariate signals?

We can just use multivariate convolution,

$$\underbrace{X[t]}_{\in\mathbb{R}^P} = \sum_{k=1}^{K} \left(z_k * D_k\right)[t] = \sum_{k=1}^{K}\sum_{\tau=1}^{L} z_k[t-\tau]\,\underbrace{D_k[\tau]}_{\in\mathbb{R}^P}$$

with:

- X a multivariate signal of length T in $\mathbb{R}^P$
- D_k a multivariate signal of length L in $\mathbb{R}^P$
- z_k a univariate activation signal of length $\widetilde{T} = T - L + 1$

However, this model does not account for the physics of the problem.

Rank-1 constrained dictionary learning

References

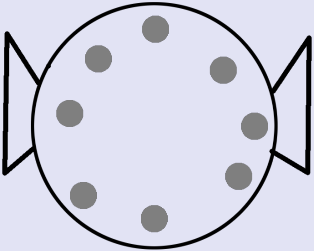
- Dupré la Tour, T., Moreau, T., Jas, M., and Gramfort, A. (2018).

  Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals.

  In *Advances in Neural Information Processing Systems (NeurIPS)*, pages 3296–3306, Montreal, Canada

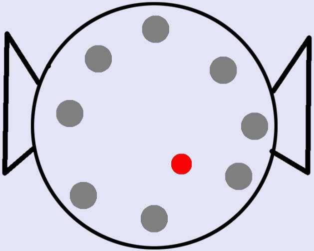
EM wave diffusion

- Recording here with 8 sensors



EM wave diffusion

- Recording here with 8 sensors
- EM activity in the brain



EM wave diffusion

- Recording here with 8 sensors
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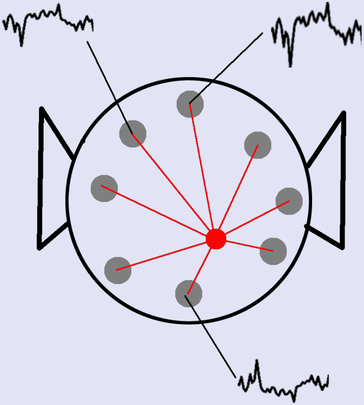
- The electric field is spread **linearly** and **instantaneously** over all sensors (Maxwell equations)



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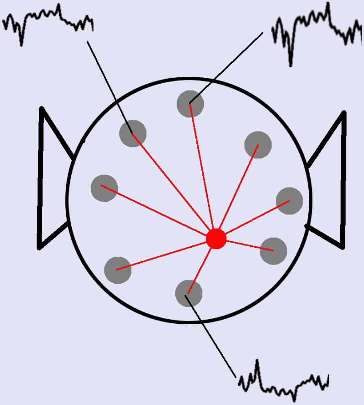
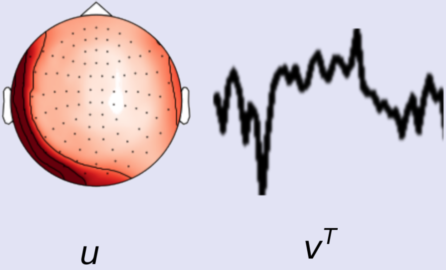




Multivariate CSC with rank-1 constraint

Idea: Impose a rank-1 constraint on each dictionary atom D_k

To make the problem tractable, use u_k and v_k *s.t.* $D_k = u_k v_k^\top$.

$$\min_{u_k, v_k, z_k^n} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * (u_k v_k^\top) \right\|_2^2 + \lambda \sum_{k=1}^{K} \left\| z_k^n \right\|_1 , \qquad (1)$$
$$\text{s.t.} \quad \|u_k\|_2^2 \leq 1 \text{ , } \|v_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0 \text{ .}$$

Here,

- $u_k \in \mathbb{R}^P$ is a spatial pattern
- $v_k \in \mathbb{R}^L$ is a temporal pattern

$\Rightarrow$ This is a tri-convex problem

Z-step: Locally greedy coordinate descent (LGCD)

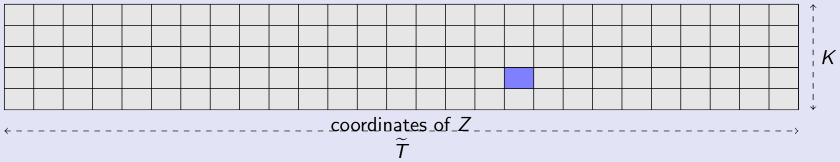
Coordinate Descent: only 1 coordinate is updated at each iteration:

1. The coordinate $z_{k_0}[t_0]$ is updated to its optimal value $z'_{k_0}[t_0]$ when all other coordinate are fixed.

2. The updated coordinate is chosen

   - Cyclic: $\mathcal{O}(1)$ [Friedman et al., 2007]
   - Randomized: $\mathcal{O}(1)$ [Nesterov, 2010]
   - Greedy: $\mathcal{O}(K\widetilde{T})$ [Osher and Li, 2009]
     by maximizing $|z_k[t] - z'_k[t]|$
   - Locally Greedy: $\mathcal{O}(K\widetilde{L})$ [Moreau et al., 2018]
     by maximizing $|z_k[t] - z'_k[t]|$ on a window

We introduced the LGCD method which is an extension of GCD.



GCD has $\mathcal{O}(K\widetilde{T})$ computational complexity.

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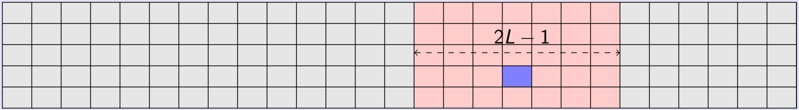

coordinates of Z

GCD has $\mathcal{O}(K\widetilde{T})$ computational complexity.

But the update itself has complexity $\mathcal{O}(KL)$

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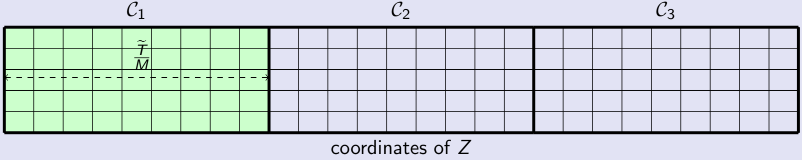

With a partition $\mathcal{C}_m$ of the signal domain $[1, K] \times [0, \widetilde{T}[$,

$$\mathcal{C}_m = [1, K] \times [\frac{(m-1)\widetilde{T}}{M}, \frac{m\widetilde{T}}{M}[$$

Locally Greedy Coordinate Descent [Moreau et al., 2018]

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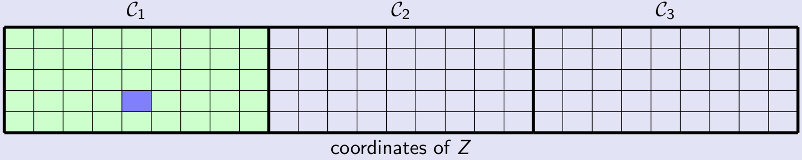

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The coordinate to update is chosen greedily on a sub-domain $\mathcal{C}_m$

$$\frac{\widetilde{T}}{M} = 2L - 1 \quad \Rightarrow \quad \mathcal{O}(\text{Coordinate selection}) = \mathcal{O}(\text{Coordinate Update})$$

The overall iteration complexity is $\mathcal{O}(KL)$ instead of $\mathcal{O}(K\widetilde{T})$.

$\Rightarrow$ Efficient for sparse Z

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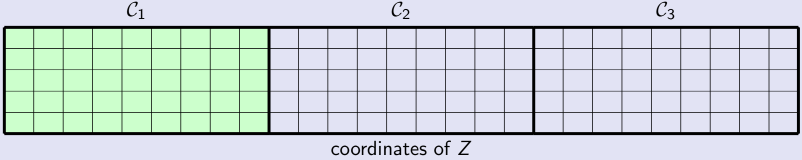

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$\Rightarrow$ Efficient for sparse Z $\Rightarrow$ Can be efficiently parallelized.

D-step: solving for the atoms

We use the projected gradient descent with an Armijo backtracking line-search Wright and Nocedal [1999] for both u-step and v-step for

$$\min_{\substack{\|u_k\|_2 \leq 1 \\ \|v_k\|_2 \leq 1}} E(u_k, v_k) \triangleq \sum_{n=1}^{N} \frac{1}{2} \|X^n - \sum_{k=1}^{K} z_k^n * (u_k v_k^\top)\|_2^2 \quad . \tag{2}$$

One important computation trick is for fast computation of the gradient.

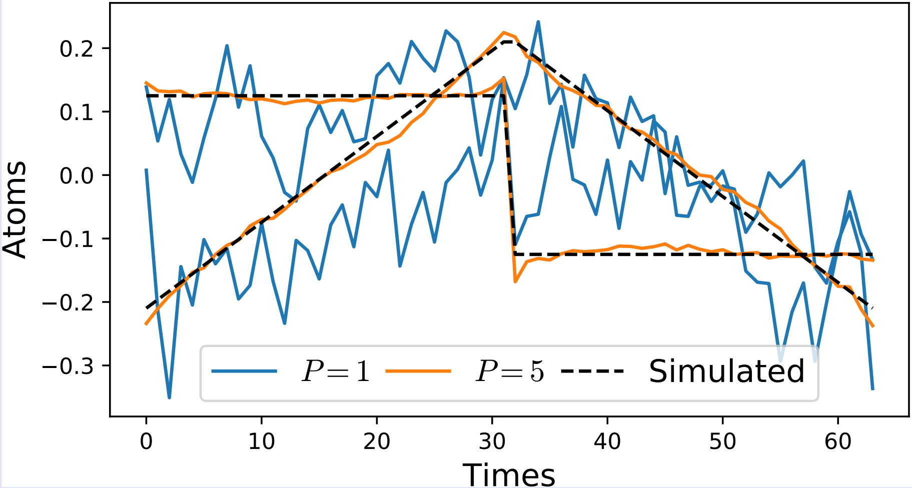
$$\nabla_{u_k} E(u_k, v_k) = \nabla_{D_k} E(u_k, v_k) v_k \quad \in \mathbb{R}^P ,$$
$$\nabla_{v_k} E(u_k, v_k) = u_k^\top \nabla_{D_k} E(u_k, v_k) \quad \in \mathbb{R}^L ,$$

Computing $\nabla_{D_k} E(u_k, v_k)$ can be done efficiently

$$\nabla_{D_k} E(u_k, v_k) = \sum_{n=1}^{N} (z_k^n)^{\Lsh} * \left(X^n - \sum_{l=1}^{K} z_l^n * D_l \right) = \Phi_k - \sum_{l=1}^{K} \Psi_{k,l} * D_l \ ,$$

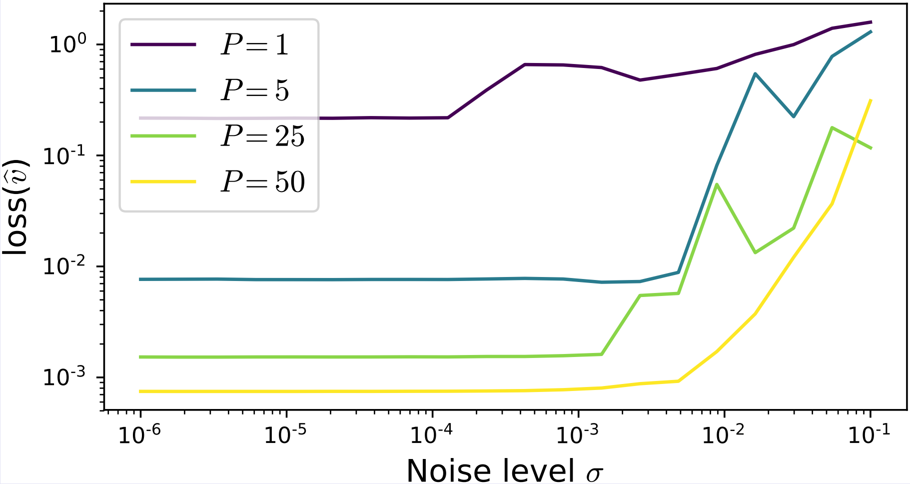
Pattern recovery

Patterns recovered with $P = 1$ and $P = 5$. The signals were generated with the two simulated temporal patterns and with $\sigma = 10^{-3}$.



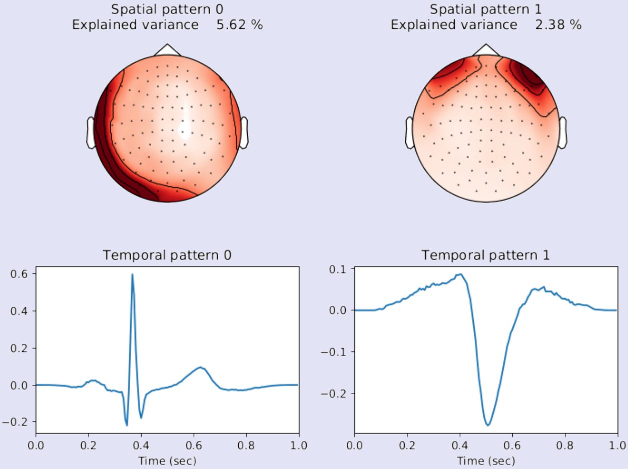
Pattern recovery

Evolution of the recovery loss with σ for different values of P. Using more channels improves the recovery of the original patterns.

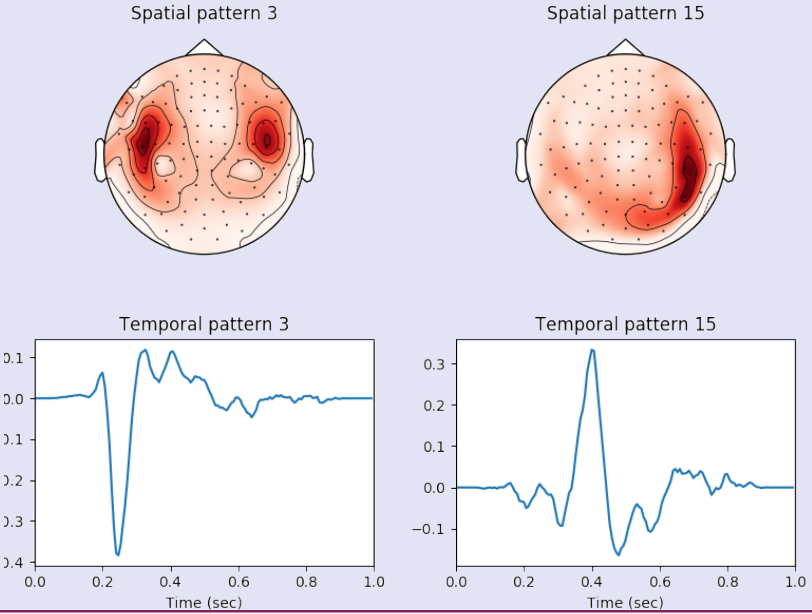


MNE sample data

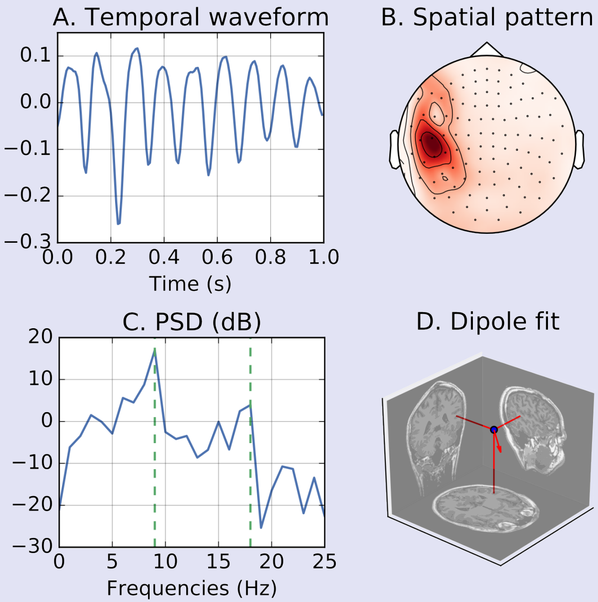
A selection of temporal waveforms of the atoms learned on the MNE sample dataset.



Learned atoms – Evoked response



Learned atoms – Induced responses



alphacsc Model descriptions Examples Gallery API Documentation 0.4.0 (stable)

Search the docs ...

alphaCSC: Convolution sparse coding for time-series

unittests passing codecov 82%

On this page

This is a library to perform shift-invariant sparse dictionary lea
(CSC), on time-series data. It includes a number of different r

1. univariate CSC
2. multivariate CSC
3. multivariate CSC with a rank-1 constraint [1]
4. univariate CSC with an alpha-stable distribution [2]

A mathematical descriptions of these models is available in th

Python code online:
https://alphacsc.github.io

```
pip install alphacsc
```

Installation

To install this package, the easiest way is using `pip`. It will instal
`setup.py` depends on `numpy` and `cython` for the installation s
this package, please run one of the two commands:

Examples reproduce figures from this talk!

(Latest stable version)

```
pip install alphacsc
```

(Development version)

```
pip install git+https://github.com/alphacsc/alphacsc.git#egg=alphacsc
```

(Dicodile backend)

```
pip install numpy cython
pip install alphacsc[dicodile]
```

Modeling stimuli induced patterns with Point Processes

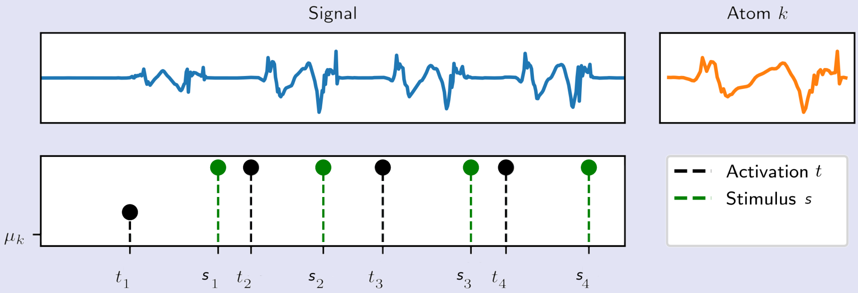
References

- Allain, C., Gramfort, A., and Moreau, T. (2022). DriPP: Driven Point Process to Model Stimuli Induced Patterns in M/EEF Signals.

  In *International Conference on Learning Representations (ICLR)*

Stimuli Induced Patterns

- Manual pattern identification
- No quantification of how stimuli influence patterns activation.



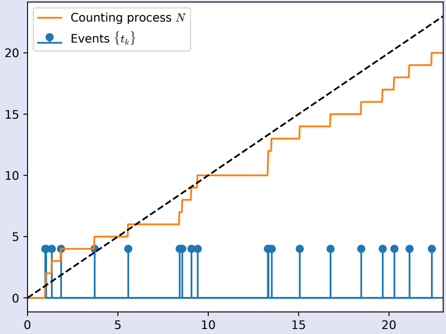
Activations and stimuli can be seen as *Point Processes*.

- Stochastic model for stream of events
- Time of arrival $\{t_k\}$ associated with counting process $N(t)$
- Characterized by the intensity:

$$\lambda(t|\mathcal{F}_t) = \lim_{dt \to 0} \frac{P(N(t+dt) - N(t) = 1|\mathcal{F}_t)}{dt}$$

Poisson process with constant probability of arrival

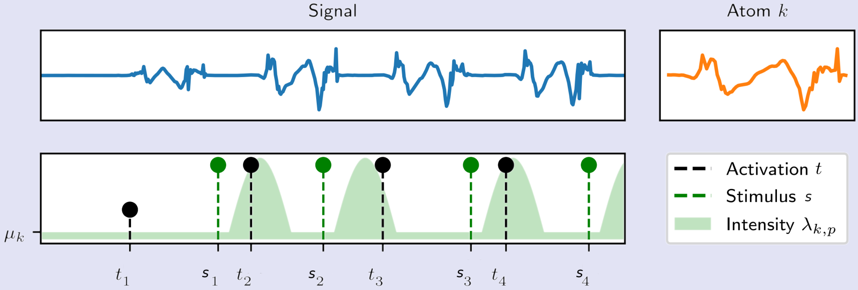
$$\lambda(t) = \mu_0$$



DriPP – Driven Point Process

Idea: Model the intensity of the activation $\{t_k\}$ depending on the PP from the stimuli $\{s_p\}$.

$$\lambda(t|\mathcal{F}_t) = \lambda(t|\{s_p; s_p < t\}) = \mu_0 + \sum_{s_p < t} \kappa(t - s_p)$$

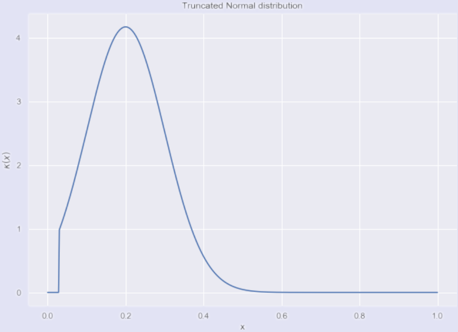


Modeling latency

Chosing a model for stimuli based modeling:

$$\lambda(t|\mathcal{F}_t) = \mu_0 + \sum_{s_p<t} \alpha\kappa(t - s_p)$$

- $\mu_0 \geq 0$: spontaneous activity.
- $\alpha \geq 0$: allow for stimuli to have no effect.
- $\kappa(\tau)$: pdf of a truncated Gaussian $\mathcal{N}(m, \sigma^2)$ to model latency.



Parameters estimation

The negative log-likelihood of the model can be computed using the intensity λ:

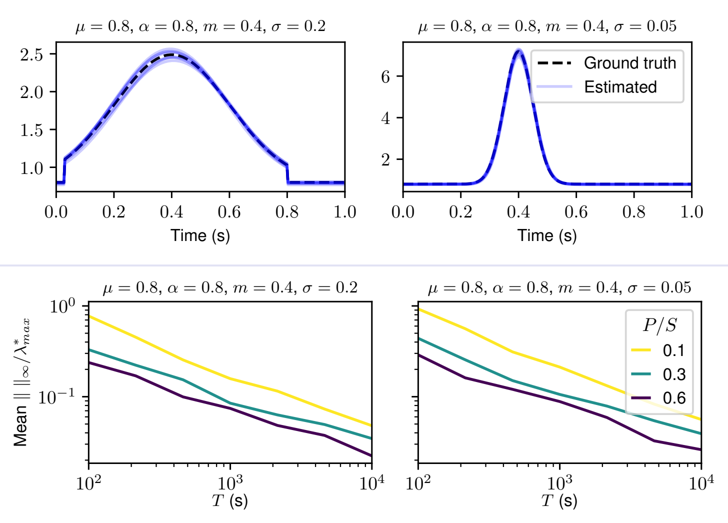
$$\begin{aligned}\mathcal{L}(\{t_k\};\Theta) &= \int_0^T \lambda(t)dt - \sum_{t_k} \lambda(t_k) \\ &= \mu_0 T + \alpha|\{t_k\}| - \sum_{t_k} \log(\mu_0 \sum_{s_p<t_k} \alpha\kappa(t_k - s_p))\end{aligned}$$

with $\Theta = (\mu_0, \alpha, m, \sigma^2)$

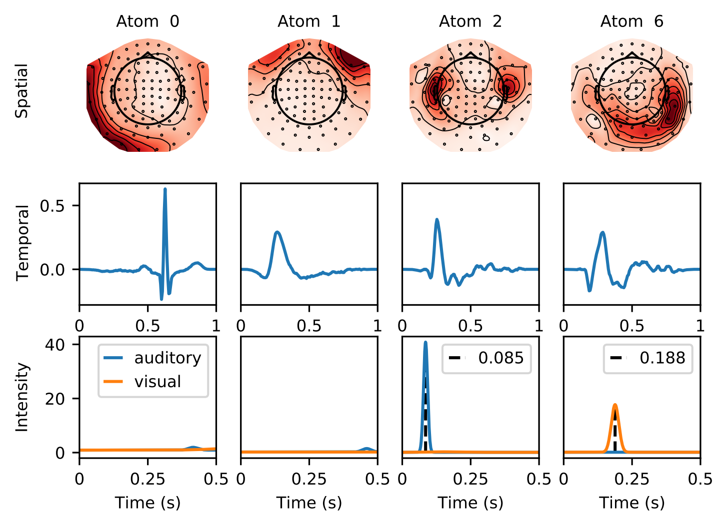
$\Rightarrow$ Parameter estimation is done using an EM algorithm.

Parameters recovery

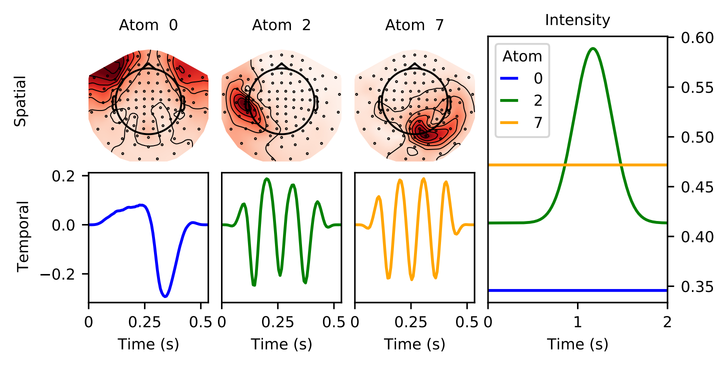
Events simulated with the Truncated Gaussian model:



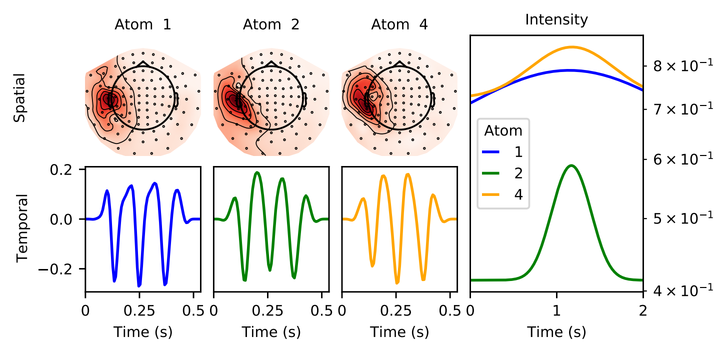
Results for artifacts and evoked atoms - samples



Results for artifacts and evoked atoms - somato



Results for artifacts and evoked atoms - somato



Conclusion

- CDL can learn recurring patterns in multivariate signals.
- Converts the signal into a stream of events.
- PP framework can model the activation distribution.

Limitations and on-going work:

- Not easy to apply to population level.
- DriPP does not model inhibition.
- CDL and PP are separated.

Thanks for your attention!

Code available online:

alphacsc : alphacsc.github.io

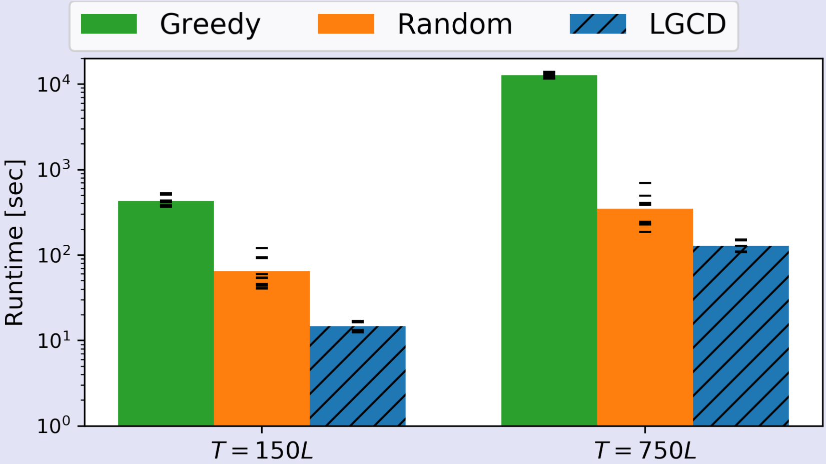
DriPP : github.com/CedricAllain/dripp

Slides are on my web page:

tommoral.github.io @tomamoral

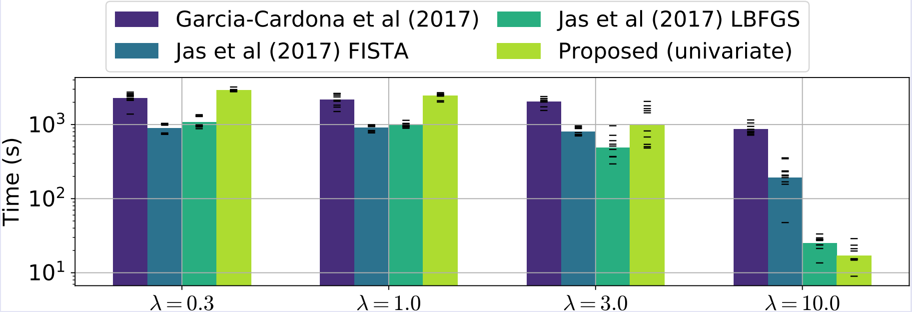
Fast optimization

Comparison of the coordinate selection strategy for CD on simulated signals
We set $K = 10$, $L = 150$, $\lambda = 0.1\lambda_{\max}$



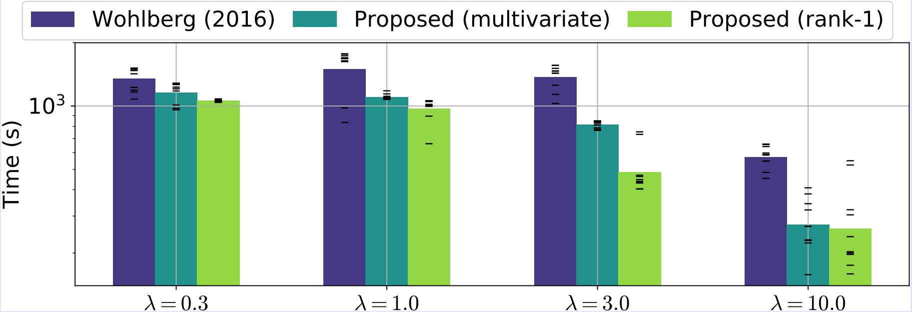
Fast optimization

Comparison with univariate methods on somato dataset with $T = 134,700$, $K = 8$ and $L = 128$



Fast optimization

Comparison with multivariate methods on somato dataset with $T = 134,700$, $K = 8$, $P = 5$ and $L = 128$



Good scaling in the number of channels P

Scaling relative to P on somato dataset with $T = 134,700$, $K = 2$, and $L = 128$